Control of Thermal Impact for Thermal Safety

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The prime interest of this study is to minimize the thermal impact of a hot surface on human tissue. Since thermal burn occurs when the physiological reaction to the thermal impact is beyond the limit of living cells' resistance, a safe level of thermal potential may be controlled by means of physical processes. An analytical solution of the linear heat conduction equation as applied to a three-layer model is obtained under a given selected set of initial and boundary conditions. The parameters derived from the solution are discussed and subjected to appropriate experiments. The results verify the assumed conditions.

Nomenclature

A, B, and C	= integral constants
a	= insulating layer thickness, cm
c	= volume heat capacity, cal·g^{-1}·°C, (cal = 4.1840J)
k	= thermal conductivity cal·cm^{-1}·sec^{-1}·°C^{-1}
L^{-1}	= inverse Laplace transform operator
n	= running index
P	= Laplace transform constant, $P>0$
q	= parameter, defined in Eq. (A1)
R	= defined in Eq. (A11b)
U	= defined in Eq. (A11c)
T	= temperature, °C
t	= time, sec.
x	= coordinate, length, cm
y,z	= thermal inertia fraction, defined in Eq. (A11b)
Y,Z	= combination of thermal inertia fractions, defined in Eq. (A11b)
α	= thermal diffusivity, cm^2·sec^{-1}
θ	= scale adjusted temperature, °C, refers to initial skin temperature, T_0
λ	= thermal inertia ($\lambda \equiv k\rho c$), cal^2·cm^{-4}·sec^{-1}·°C^{-2}
μ	= defined in Eq. (A11c)
ρ	= density, g·cm^{-3}
τ	= dimensionless time, defined in Eq. (13)
ϕ	= defined in Eq. (A11c)
Subscripts	
c	= refers to the initial hot side constant temperature
i	= refers to layers, 1, 2, or 3
0	= refers to the initial skin temperature
s	= refers to surface

Introduction

A BURN may result when one contacts a hot surface. The severity of such a burn will be determined by two processes, one physiological and the other physical. The former is a function of the physiological reaction of living cells to thermal impact, which is the rate at which thermal energy is transmitted to the human tissue from the hot surface. The latter is a function of the thermal potential that exists at the hot surface, which is the temperature drop across a thermal field. Once the severity of cell damage due to thermal impact in the physiological process is determined,[1] criteria can be developed for keeping the thermal potential of hot surfaces at a safe level. This is accomplished by altering the thermal properties of the surface material in accordance with the physical principle of heat conduction.[2]

It is well known that whenever the temperature of cells in the epidermal layer of human skin is raised above 44 °C, cell destruction is initiated.[3] The higher the tissue temperature becomes, the shorter the time required for cell damage to become irreversible. For a given material and surface temperature, the maximum contact time can be determined beyond which thermal injury will occur. This simple time-temperature relationship has been discussed in previous studies[1,4] and is reproduced in Fig. 1.

In Fig. 1, the time-temperature thresholds at which cutaneous burning occurs are shown. The solid line indicates threshold A at which epidermal necrosis of porcine skin occurs. The data points are from Henriques' results of critical experimental exposures of human skin in vivo.[1] They represent the shortest exposure (contact) time at the indicated temperature for complete destruction of epidermis. The broken line indicates threshold B, the longest exposure (contact) time resulting in reversible epidermal injury.

The thermal impact to human tissue, exerted by a hot surface, can be described by means of a linear Fourier heat conduction equation, as was shown in the previous study.[4] In that study a simple model having an assumed linear heat flow through a semi-infinite composite solid was used. In the present study methods are examined for minimizing the thermal impact of hot surfaces to human tissue by varying the thermal properties of the hot surface.

According to the heat flow equation, the rate of thermal energy received by human tissue from a contacted hot surface is a function of the temperature and the thermal properties of that surface. The temperature of the surface in turn is a function of the heat source, the thermal properties of the medium through which the heat flows, and the distance between the heat source and the surface. Therefore, when human tissue is in contact with a surface of a hot object and the thermal properties of that surface are the same as those of

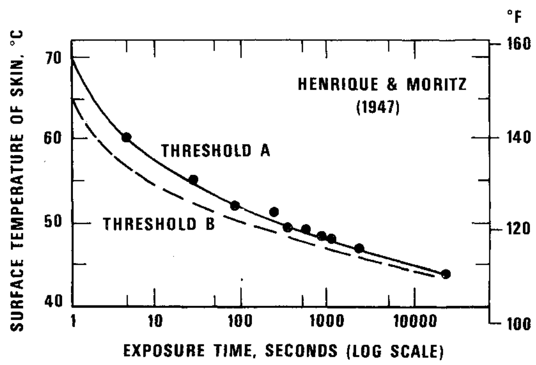

Fig. 1 Temperature-time relationship in burns.

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the heated medium, the problem of heat flow will be the same as that of heat conduction in a semi-infinite composite solid as described in the simple model in Ref. 4. On the other hand, if the thermal properties of the surface and the medium are not the same, then heat will flow through two different media before it reaches human tissue. Then a three-layer model will be needed to analyze the problem. The temperature at the surface is modified by the thermal properties of the surface layer. Thus by proper selection of surface materials, thermal impact can be controlled and thermal injury minimized.

In order to explore the effects of surface modification on thermal impact, an analytical solution is sought to a linear heat conduction equation representing a three layer model. Because of the complexity of the mathematics involved, simple initial and boundary conditions are selected so that the physical meaning of the solution in terms of practical applications can be easily understood. The parameters derived from the solution will be examined and compared with appropriate experiments.

Conductive Heat Transfer in Multilayer Solids

The principle of conductive heat transfer in a multilayer solid has been well established.[5] For simplicity, we shall take the heat transfer process as a linear conduction through three different solid layers. The thermal properties of each layer are assumed to be different from each other. Figure 2 illustrates the situation.

For practical purposes, we shall consider region 1 to be steel, region 2 to be an insulating layer, such as enameled porcelain or plastic, and region 3 to be human tissue. Because steel is adjacent to the heated source, in home applicances, we shall assume it is a semi-infinite body where a thermal steady state is reached. Since the insulating layer is chemically bound to the steel plate, it is assumed that there is no contact resistance between them, and that this layer is homogeneous so that the temperature is uniform and is equal to T_c at the start. Finally, the human tissue is assumed to be an homogeneous semi-infinite body, and in perfect contact with the surface of the insulating layer.

The heat conduction equation for the system is:

$$\rho_i c_i \partial T_i / \partial t = k_i \partial^2 T_i / \partial x^2 \qquad (1)$$

$i = 1, 2$ or 3: when $x < 0$, $i = 1$; $0 < x < a$, $i = 2$; or $x > a$, $i = 3$

The initial conditions are:

$$T_1 = T_2 = T_c, \text{ and } T_3 = T_0, \text{ when } t = 0 \qquad (2)$$

The boundary conditions are:

$$T_1 = T_2;\; k_1 \partial T_1 / \partial x = k_2 \partial T_2 / \partial x, \text{ when } x = 0;\; t > 0 \qquad (3)$$

$$T_2 = T_3;\; k_2 \partial T_2 / \partial x = k_3 \partial T_3 / \partial x, \text{ when } x = a;\; t > 0, \text{ and}$$

$$\lim_{x \to -\infty} T_1 = T_c;\; \lim_{x \to +\infty} T_3 = T_0 \qquad (4)$$

The solutions to this system follow and are derived in the Appendix (all symbols are derived therein):

Let $\theta_i = T_i - T_0$, then

$$\theta_1 = \theta_c \left[1 - 2yz \sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]\mu a - x}{2\sqrt{a_1 t}} \right) \right] \qquad (5)$$

$$\theta_2 = \theta_c \left[1 - z \sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]a - x}{2\sqrt{a_2 t}} \right) \right.$$

$$\left. + Yz \sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]a + x}{2\sqrt{a_2 t}} \right) \right] \qquad (6)$$

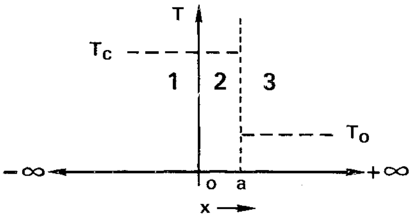

Fig. 2 Linear heat conduction through a three-layer solid.

$$\theta_3 = \theta_c (1 - z) \left[\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{x - a.2n\phi a}{2\sqrt{a_3 t}} \right) \right.$$

$$\left. + Y \sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{x - a + 2[2+1]\phi a}{2\sqrt{a_3 t}} \right) \right] \qquad (7)$$

Physical Aspects of the Model

The mathematics developed in the last section are intended to establish the functional relationships and to show which parameters may be controlled for the attainment of thermal safety. The temperature distribution for each region are established as shown by Eqs. (5-7). The relative temperature, θ_i / θ_c, as usual, is a function of the thermal inertia, $\lambda^{1/2} = (k\rho c)^{1/2}$, fractions and their combinations.[2,5] Because all the inertia fractions are smaller than unity, and the complimentary error function takes values from 1 to 0 and approaches zero as n increases, therefore, all the infinite series in the solution are convergent. For a given time, the convergence of the infinite series depends on the values of **R** and a. The smaller the value of **R** or the larger the value of a, the more rapid is the convergence. On the other hand, if t is very large, the error function approaches zero. In which case the temperature distribution will depend on the thermal inertia of the two semi-infinite blocks, as if the second layer were not in existence. Thus, $\theta_1 = \theta_2 = \theta_3 = \theta_c \lambda_1^{1/2} / (\lambda_1^{1/2} + \lambda_3^{1/2})$ which is identical to the surface contact temperature, T_s, and is constant[5] as shown in the previous study,[4]

$$T_s = T_a - (T_a - T_b) / [1 + (\lambda_a / \lambda_b)^{1/2}] \qquad (8)$$

where

$$T_a = T_c,\; T_b = T_0,\; (\lambda_a)^{1/2} = (\lambda_1)^{1/2} \text{ and } (\lambda_b)^{1/2} = (\lambda_3)^{1/2}$$

Therefore, the influence of the second layer on the temperature is meaningful only when the contact time is reasonably short. The significance of this is that under certain conditions of contact between human tissue and a hot surface, the onset of irreversible cell damage in the tissue can be prevented by reducing thermal impact; this gives the human subject time to react to the sensation of heat and to terminate contact with the hot surface.

On the examination of Eq. (6), the parameters that affect the temperature distribution are the time, the thickness, and the thermal properties of the material. Once these parameters are under control the thermal impact from a hot surface may be minimized. The time and thickness may be controlled by physical means; however, the desired values of thermal properties of material may not be controlled, but must be suitably chosen. If these properties are properly selected, the thermal impact as the rate of thermal energy transmission may then be expressed in terms of temperature gradient and the rate of temperature change at the surface in contact with human skin.

According to the system described in Fig. 2, the surface in contact with human skin is at $x = a$, and the temperature is θ_2. Thus, according to Eq. (6), the rate of temperature change is:

$$-\frac{\partial}{\partial t} (\theta_2 / \theta_c) \big|_{x=a} = 2z \left(1 + \frac{1}{Z} \right) \frac{a}{\sqrt{\pi \alpha_2 t^3}} \sum_1^\infty n \mathbf{R}^n e^{-\frac{n^2 a^2}{\alpha_2 t}} \qquad (9)$$

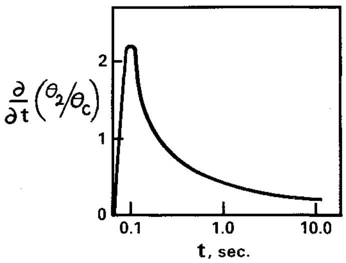

Fig. 3 Rate of thermal energy dissipation to human skin from the surface of porcelain (0.01cm) enameled steel.

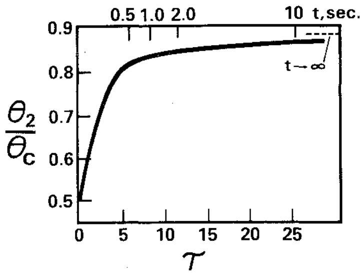

Fig. 4 Temperature distribution at the surface of 0.01cm porcelain enameled steel at various values of τ.

and the temperature gradient is:

$$-\frac{\partial}{\partial x}(\theta_2/\theta_c)\,|_{x=a}=\frac{z}{\sqrt{\pi\alpha_2 t}}\left(1-\frac{2z}{Z}\sum_1^{\infty}\mathbf{R}^n e^{-\frac{n^2a^2}{\alpha_2 t}}\right) \qquad (10)$$

The last equation shows a continuous temperature drop as the thickness increases and reaches the limit of $z/(\pi\alpha_2 t)^{1/2}$. However, Eq. (9) exhibits a maximum as the time changes. Therefore, taking the first derivative with respect to t and equating it to zero yields:

$$3\sum_1^{\infty} n\mathbf{R}^n e^{-\frac{n^2a^2}{\alpha_2 t}}-\frac{2a^2}{\alpha_2 t}\sum_1^{\infty} n^3\mathbf{R}^n e^{-\frac{n^2a^2}{\alpha_2 t}}=0 \qquad (11)$$

As the series converges very rapidly for decreasing t, for sufficiently small times, the first term is used as a first approximation. We obtain

$$a/(\alpha_2 t)^{1/2}\,|_{\max}\simeq 1.225 \qquad (12)$$

It appears that for a given material with given thickness, the maximum rate of dissipation depends on the time interval. Once the energy dissipation has passed the maximum time required, the rate of dissipation will drop very rapidly and level off at a longer time. Figure 3 illustrates this point. Furthermore, if we plot $(\theta_2/\theta_c)_{x=a}$ vs τ, i.e., at the surface of the insulated layer, the leveling effect becomes more apparent (Fig. 4), where τ is a dimensionless number and is defined as

$$1/\tau=a/(\alpha_2 t)^{1/2} \qquad (13)$$

It is seen that in Fig. 4, for θ_2/θ_c at $x=a$, the surface temperature of the insulating layer is an asymptotic function of time and approaches unity as time approaches infinity. However, the ratio of temperature change is about 3% from $\tau=12$ to $\tau=25$ (2 to 10 sec). Prolonging the contact time to approach a steady state as shown by Eq. (8), the ratio of temperature change is only about 4% as compared with that at $\tau=12$ ($t\simeq 2$ sec). Therefore, for all practical purposes, Eq. (8) may be considered applicable for the interfacial temperature between human tissue and the insulating layer when they are in contact with each other for more than two seconds.

In view of the above considerations, the thermal impact is implicitly a function of three parameters for $\tau<10$: time, thickness, and the thermal properties of a chosen material. Its effect, therefore, may be determined by measuring the interfacial temperature between human tissue and the insulating layer at various thicknesses.

Experiments

Samples of porcelain enameled steel plate were provided by the Association of Home Appliance Manufacturers. The plates were of 0.088 cm. thick steel with porcelain enamel in thicknesses of 0.015, 0.020, 0.025, and 0.040 cm bonded to one side. Enamel thickness varied about ±5% from the nominal value for individual samples. Test plates of 7.0 cm square were cut from the material provided. The porcelain enamel was removed from one half of each test plate and thermocouples were attached to the surfaces of porcelain enamel and the bared steel.

A laboratory hot plate with variable temperature control was used as a heat source. For one test condition the test plates were placed in direct contact with the hot plate surface. For two other test conditions, intended to simulate kitchen ranges, a 6×6 cm square well was formed on the hot plate surface using four 7×7 cm ceramic bricks. This well was filled with glass wool for one test condition and with a solid piece of cork for another condition. Thermocouples were installed in the glass wool and in the cork. The setup is shown schematically as in Fig. 5a.

In one series of tests, volunteer subjects held their fingers in contact with the test plates for two to three seconds. A cardboard cylindrical tube of 2 cm diameter placed around the test finger held a thermocouple of 0.02 cm copper-constantan type in position on the finger tip as shown schematically in Fig. 5b. When the test finger was placed in contact with the test plate, the thermocouple was within the area of contact. In another series of tests, a thermesthesiometer designed and constructed by Marzetta of NBS[6] was used instead of a finger. The thermesthesiometer is essentially a simulated finger having thermal properties similar to those of a finger and providing a measure of the epidermal basal temperature equivalent to θ_3 at $x=a+0.01$ cm in Eq. (7). The contact time for the thermesthesiometer was 1 second.

Results and Analysis

There were two kinds of measurement: 1) the interfacial temperature between a human finger and the hot surface; 2) the temperature at the epidermal basal layer of a simulated human finger. The general pattern of measured interfacial temperature, T_s, appeared schematically as shown in Fig. 6. The mean value for T_s was recorded. The uncertainty of measurement was ±1 °C, and the statistical fluctuation of four measurements for each test was also 1 °C. The measurement with the thermesthesiometer was not a continuous recording, because the instrument was originally designed for various preset times with digital temperature reading. Three readings for each test were recorded. The overall uncertainty was ±2 °C. Results of the measurements are listed in Tables 1 and 2.

In order to compare the experimental results with the theoretical calculation, the following assumptions for both the experiment and the model are necessary: 1) the time for testing must be short, so that the condition for a semi-infinite body is met; 2) the contact resistance in the experimental measurements is neglected; and 3) the heat source, i.e., the

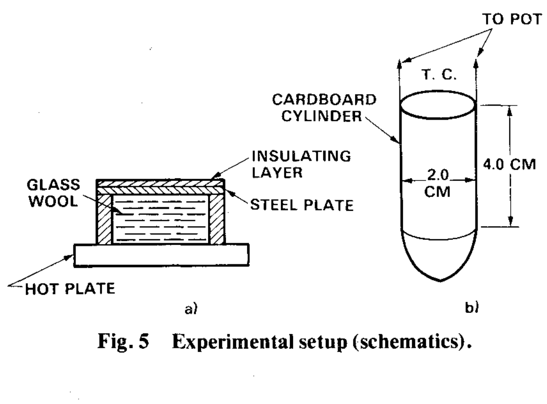

Fig. 5 Experimental setup (schematics).

Table 1 Interfacial temperatures of human tissue in contact with a hot surface of steel and porcelain enameled steel plates

Surface materials	T_{initial}	$\theta_c = T_c - T_0$	T_s	$\theta_s = T_s - T_0$	θ_s/θ
Plates coated with silicon grease directly on top of an electrically heated hot-plate					
Human tissue, finger (in vivo)	30.5				
Steel (bare)	72.0	41.5	68.8	38.3	0.92
Porcelain enamel 0.01 cm			66.2	35.7	0.86
0.02			63.0	32.5	0.78
0.04			58.0	27.5	0.66
Plates on top of a ceramic well filled with glass wool on an electrically heated hot-plate					
Human tissue, finger (in vivo)	30.0				
Steel (bare)	57.0	27.0	53.5	23.5	0.87
Porcelain enamel 0.01 cm			52.0	22.0	0.81
0.02			49.5	19.5	0.72
0.025			47.5	17.5	0.65
0.04			46.5	16.5	0.61

Table 2 Temperature for the thermesthesiometer in contact with a hot surface of steel and porcelain enameled steel plates [a]

Surface materials	T_{initial}	$\theta_c = T_c - T_0$	T_3(0.01 cm)	θ_3	θ_3/θ_c
Plates rested on cork of 7 × 7 × 0.3 cm plate which was put on top of an electrically heated hot-plate, contact time was 1 sec					
Thermesthesiometer	33				
Cork	70	37			
Steel			55.5	22.5	0.60
Porcelain enameled steel 0.01 cm			55.4	22.4	0.61
0.015			53.4	20.3	0.55
0.02			53.9	20.9	0.56
0.025			53.5	20.5	0.55
0.04			51.3	18.3	0.50
Plates directly on top of an electrically heated hot-plate, thermesthesiometer contact time was 1.0 sec					
Thermesthesiometer	33				
Steel	70	37	53.9	20.9	0.56
Porcelain enameled steel 0.01 cm			53.7	20.7	0.55
0.04			51.5	18.5	0.50

[a]The experimental results were made available by the courtesy of L. Marzetta of our laboratory.[7]

hot-plate in the experiment, is in perfect contact with the first layer of material. Thus we will assume that the heat in the first layer is uniform, so that the first layer can be considered as a semi-infinite body.

Based on all of these assumptions, and the thermal properties listed in Table 3, the interfacial temperatures of the insulated layers and human tissue, and the temperatures at the depth of an epidermal layer of human tissue are calculated in accordance with Eqs. (6) and (7). The computed results and the experimental observations are shown in Fig. 7 and 8.

From a comparison of the values of observations with those of computation, it appears that one set of experimental results, as indicated by o in Fig. 7, is in agreement within the range of experimental uncertainty, while the other, denoted by Δ, is generally lower than the computed values, especially those for the thicker insulation. Apparently, when the plate is put directly on top of the hot-plate, the experimental conditions are more compatible with the model. When glass wool is placed between the test plate and the hot-plate, however, the surface temperatures are generally lower by a degree or so beyond the uncertainty. This is because the deviation from the semi-infinite body assumption begins to appear.

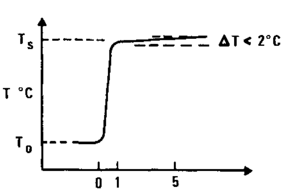

Fig. 6 Schematic shape of measurements of interfacial temperatures.

The experimental values in Fig. 8 exhibit somewhat greater departure from the model of steel-porcelain-skin. They are generally lower by a degree or two beyond the uncertainty. The departure from the computed values may be attributed to the contact resistance.

Generally speaking, our experimental setup, wherein the surface temperature being influenced by the thickness of an insulated layer is examined, is qualitative in nature. In addition, the influence of an insulation on the surface temperature is revealed by the comparison of the temperature at $x = 0$ with that at $x = a$. The overall experimental measurement of temperature may have been off by a degree or two, but the behavior of the system is well-characterized.

Table 3 Thermal properties of selected materials

Materials	Thermal conductivity $k \times 10^3$	Density ρ	Heat capacity c	Thermal diffusivity $\alpha X10^3$	Thermal inertia $\sqrt{\lambda}$
Skin[8] (epidermal)	1.3	0.9	1.1	1.313	0.03587
Steel[8]	108.	7.8	0.11	125.9	0.3044
Plastics[8]	0.59	1.28	0.37	1.25	0.01672
Borosilicates[8]	2.9	2.2	0.2	6.59	0.03572
Porcelain[a] enamels	2.9	2.2	0.2	6.59	0.03572
Cork[11]	0.1	0.13	0.48	1.6	0.0025

[a]The constituents of porcelain enamel vary widely, which depends on the purpose of application and the enameling process. Generally, the composition of porcelain enamel consists of 80-90% of borosilicates of sodium, potassium, and calcium; and 10-20% of Al_2O_3, and some kinds of metal oxides.[9,10] Obviously, there is no fixed value for thermal properties, the values chosen for this study are taken as those for sodium borosilicate.

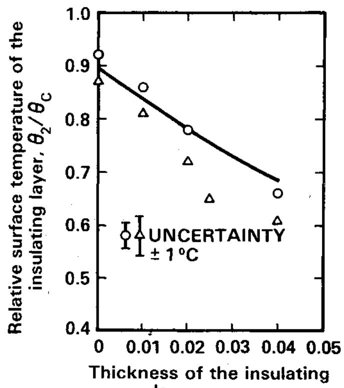

Fig. 7 Interfacial temperatures of human tissue in contact with a hot surface of steel and porcelain enameled steel plates. Solid line is calculated values based on Eq. (6). o and Δ are experimental values for plates on top of a hot-plate directly, and on top of a ceramic well filled with glass wool on a hot-plate, respectively.

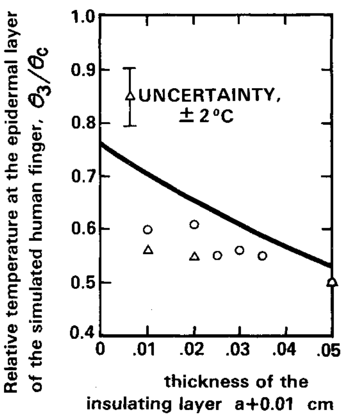

Fig. 8 Temperatures for the thermesthesiometer in contact with a hot surface of steel and porcelain enameled steel plates. The solid line indicates the calculation of Eq. (7) for the model of steel-porcelain-skin. o and Δ are the experimental values for plates on a block of cork on top of a hot-plate, and directly on top of a hot-plate, respectively.

Discussion

Model

Since the model is a mathematical idealization, an experimental deviation from such a simplification is difficult to avoid. In the model, as illustrated in Fig. 2, an infinite body with uniform thermal properties is assumed to be on the extreme left of the x-axis. The linear heat flow from left to right should, therefore, obey the one-dimensional Fourier's heat conduction equation. In actual conditions, however, the temperature distribution in the insulating layer, even though it is uniform as I_c throughout at the start, is strongly influenced by that of steel, and yet the thickness of steel in a home appliance is finite, and the temperature at this inner surface is constant, which obviously deviates from the semi-infinite solid assumed in the model. We could, of course, set up a model with a constant heat supply to flow through two layers of different thermal properties into a semi-infinite third body of another set of thermal properties, but then the mathematical solution would be too complex to be of interest for practical applications. And yet, the time interval of interest in our experiments is so short that the semi-infinite body requirement may be satisfied according to the "Law of Times".[12] The experimental results, as shown in Figs. 7 and 8, agree reasonably well in comparison with the analytical calculations. Therefore, for the scope of the study, our model is satisfactory.

The Thermal Hazard Potential

The combination of human tissue and the medium through which heat flows constitutes a thermal field. The temperature drop across such a field has been defined as thermal potential. In the area of thermal safety, the higher the thermal potential across the human tissue in a thermal field, the more severe the thermal injury to the human tissue becomes. In this context, the thermal potential is called the thermal hazard potential. Therefore, for given thermal properties, if we could control

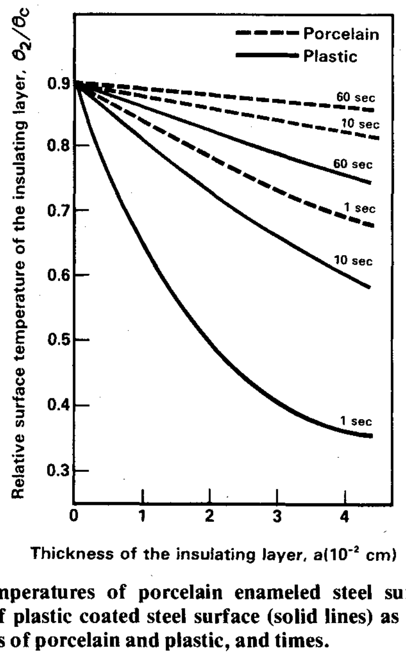

Fig. 9 Temperatures of porcelain enameled steel surface (dotted lines) and of plastic coated steel surface (solid lines) as a function of the thickness of porcelain and plastic, and times.

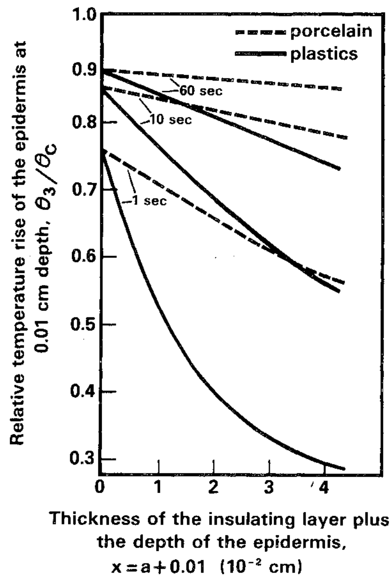

Fig. 10 Temperatures at the epidermal layer (0.01cm depth) when human skin was in contact with porcelain enameled steel (dotted lines) and plastic coated steel (solid lines) as a function of the thickness of porcelain and plastic, and time.

the temperature in a thermal field we would be able to control the thermal hazard potential.

According to Eqs. (5-7), the temperature in a thermal field is a function of three parameters, time, thermal properties, and thickness of the materials in question. In the thermal safety area, the contact time is more or less predetermined by the physchophysiological effect in casual contact and by the actual operating time requirement in necessary operational handling. The second and third parameters, thickness and thermal properties, are directly related to the materials in question. In practice, the first and the third sections in the thermal field are approximately semi-infinite. The thermal properties of one section in the field are given, those of human tissue. Those of the other sections, however, are predetermined by the chracteristics of engineering design. If the first section is the structural material, such as steel, whose thermal properties are fixed, then only those parameters in the second section can be manipulated to affect the temperature control.

The following figure (Fig. 9) obtained by plotting Eq. (6) demonstrates the effects. θ_2/θ_c is the ratio of differential temperatures at the surface, referred to the initial skin temperature, as a function of the thickness of two different insulated materials with different thermal properties at three contact times.

Thermal Safety Aspects

The temperature drop is affected by three parameters: time, thickness, and thermal properties. It appears that the combination of both thermal properties and thickness exert a great influence on thermal potential. For instance, the surface temperature of a bare steel metal at 70°C in contact with human tissue at 33°C for 1 second, as seen from Fig. 9, will be reduced approximately from 66 to 58°C if insulated with a 0.04 cm (16 mil) thickness of enameled porcelain, and from 66 to 46°C with the same thickness of plastic coating. Furthermore, when human tissue is in contact with a hot surface, the thermal hazard potential of the surface depends upon its ability to deliver thermal energy to the human tissue per unit time. The ability depends, in turn, upon the thermal conductivity and volume heat capacity of the surface material in question. As the human tissue is to receive such thermal energy, the temperature rise, θ_3, in human tissue at a depth of 100 micrometer will be an appropriate measure for comparing the influence of the three parameters of surface materials.

The plot of θ_3/θ_3 vs x of Eq. (7) demonstrates the effects. θ_3/θ_c is the ratio of the temperature rise of the epidermis at a depth of 100 μm to the initial temperature difference as a function of surface thickness for three contact times, and two surface materials: 1) porcelain (dotted lines) and 2) plastics (solid lines) in Fig. 10.

In the previous study,[4] we have established a direct relationship between the surface temperature and the thermal inertia of materials. A group of common materials chosen to illustrate the relationship was shown in Fig. 6 of that paper.[4] Through that relationship and the temperature functions illustrated in Figs. 9 and 10, it is expected that a wide range of surface materials are available for use in minimizing the thermal hazard potential of hot objects to within safety limits. Furthermore, inasmuch as the processes of coating, laminating, or enameling are well established, there should not be a problem in developing a composite material which is suitable for thermal safety.

Appendix

Let $\theta = T - T_0$, and let $\bar{\theta}$ be the Laplace transform of θ. Then, the subsidiary equation[5] of Eq. (1) is:

$$d^2\bar{\theta}_i/dx^2 - q_i^2\bar{\theta}_i = -\theta_i'/\alpha_i \tag{A1}$$

where

$q_i^2 = P/\alpha_i$; $\alpha_i = k_i/\rho_i c_i$; and $p > 0$, Laplace transform constant.

θ_i' = a constant as follows:

$$\theta_i' = \theta_2' = \theta_c; \text{ and } \theta_3' = \theta_0 = 0, \text{ when } t = 0$$

The boundary conditions for the subsidiary equations become

$$\bar{\theta}_1 = \bar{\theta}_2;\ k_1\partial\bar{\theta}_1/\partial x = k_2\partial\bar{\theta}_2/\partial x, \text{ when } x = 0 \tag{A2}$$

and

$$\bar{\theta}_2 = \bar{\theta}_3;\ k_2\partial\bar{\theta}_2/\partial x = k_3\partial\bar{\theta}_3/\partial x \tag{A3}$$

The solutions to Eq. (A1) are:

$$\bar{\theta}_1 = \frac{\theta_c}{p} + Ae^{q_1 x} \tag{A4}$$

$$\bar{\theta}_2 = \frac{\theta_c}{p} + B_1 e^{q_2 x} + B_2 e^{-q_2 x} \tag{A5}$$

$$\bar{\theta}_3 = Ce^{-q_3 x} \tag{A6}$$

where A, B_1, B_2, and C are the integration constants which may be determined by Eqs. (A2) and (A3).

Thus,

$$A = -\frac{\theta_c}{p}(1-Y)z\frac{e^{-q_2 a}}{1-Re^{-2q_2 a}} \tag{A7}$$

$$B_1 = -\frac{\theta_c}{p}z\frac{e^{-q_2 a}}{1-Re^{-2q_2 a}} \tag{A8}$$

$$B_2 = \frac{\theta_c}{p}Yz\frac{e^{-q_2 a}}{1-Re^{-2q_2 a}} \tag{A9}$$

$$C = \frac{\theta_c}{p}(1-z)e^{q_3 a}\frac{1+Ye^{-2q_2 a}}{1-Re^{-2q_2 a}} \tag{A10}$$

where the following substitutions have been made:
Ratio of thermal inertias, $\sqrt{\lambda} \equiv k/\sqrt{\alpha}$

$$\frac{k_2\, q_2}{k_1\, q_1} = \sqrt{\frac{\lambda_2}{\lambda_1}}, \quad \frac{k_3\, q_3}{k_2\, q_2} = \sqrt{\frac{\lambda_3}{\lambda_2}} \tag{A11a}$$

Thermal inertia fractions and their combinations

$$y = \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1}+\sqrt{\lambda_3}}, \qquad Y = 1-2y = \frac{\sqrt{\lambda_1}-\sqrt{\lambda_2}}{\sqrt{\lambda_1}+\sqrt{\lambda_2}} \tag{A11b}$$

$$z = \frac{\sqrt{\lambda_3}}{\sqrt{\lambda_2}+\sqrt{\lambda_3}}, \qquad Z = 1-2z = \frac{\sqrt{\lambda_2}-\sqrt{\lambda_3}}{\sqrt{\lambda_2}+\sqrt{\lambda_3}}, \quad \mathbf{R} = -YZ$$

Ratio of thermal diffusivities

$$\mu = \frac{q_2}{q_1} = \sqrt{\frac{a_1}{a_2}}, \quad \phi = \frac{q_2}{q_3} = \sqrt{\frac{a_3}{a_2}}, \quad \frac{1}{1-u} = \sum_0^\infty U^n \tag{A11c}$$

Equations (A4-A6) then become:

$$\bar{\theta}_1 = \frac{\theta_c}{p}\left\{1-2y\sum_0^\infty \mathbf{R}^n e^{-q_1[(2n+1)\mu a - x]}\right\} \tag{A12}$$

$$\bar{\theta}_2 = \frac{\theta_c}{p}\left\{1-z\sum_0^\infty \mathbf{R}^n e^{-q_2[(2n+1)a-x]} + Yz\sum_0^\infty \mathbf{R}^n e^{-q_2[(2n+1)a+x]}\right\} \tag{A13}$$

$$\bar{\theta}_3 = \frac{\theta_c}{p}(1-z)\left\{\sum_0^\infty \mathbf{R}^n e^{-q_3(x-a+2n\phi a}\right.$$

$$\left. + Y\sum_0^\infty \mathbf{R}^n e^{-q_3[x-a+2(n+1)\phi a]}\right\} \tag{A14}$$

The inversion of Laplace transform is as follows[5]:

$$\mathbf{L}^{-1}\left(\frac{1}{p}\right) = 1, \; \mathbf{L}^{-1}\left(\frac{e^{-qx}}{p}\right) = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\operatorname{erf}(s) = \frac{2}{\sqrt{\pi}}\int_0^s e^{-v^2}\,dv$$

the error function and $\operatorname{erfc}(s) = 1 - \operatorname{erf}(s)$, the complementary error function.

Then

$$\theta_1 = \theta_c\left[1-2yz\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]\mu a - x}{2\sqrt{a_1 t}}\right)\right] \tag{A15}$$

$$\theta_2 = \theta_c\left[1-z\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]a-x}{2\sqrt{a_2 t}}\right)\right.$$

$$\left. + Yz\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{[2n+1]a+x}{2\sqrt{a_2 t}}\right)\right] \tag{A16}$$

$$\theta_3 = \theta_c(1-z)\left[\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{x-a+2n\phi a}{2\sqrt{a_3 t}}\right)\right.$$

$$\left. + Y\sum_0^\infty \mathbf{R}^n \operatorname{erfc}\left(\frac{x-a+2[2+1]\phi a}{2\sqrt{a_3 t}}\right)\right] \tag{A17}$$

as shown in Eqs. (5, 6, and 7) in the text.

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